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Re-Examining Categories of Computer-Based Learning in Mathematics Education

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FRAMEWORKS FOR CLASSIFYING COMPUTER USE IN SCHOOLS

In 2002, the online journal *Contemporary Issues in Technology and Teacher Education* republished the seminal paper by Arthur Luehrmann discussing approaches to using computers in schools in the 1970s (Luehrmann, 2002a). Juxtaposed with this republished paper was a reflection on these approaches, 30 years later (Luehrmann, 2002b). His papers considered the categories of computer use in schools identified by Taylor (1980); that is, as a tutor, tool, and tutee.

In the 70s and 80s most computer software was developed as a way to *tutor* or guide individual learners. As a *tool* students use computer applications such as spreadsheets, word processors, and graphing software to help them complete inquiry-based tasks, such as problem solving and mathematical modelling. Not surprisingly, in 1980, Taylor saw this application occurring more frequently outside of schools. Students were seen to use the computer as a *tutee* when they programmed the computer to solve problems, such as in a LOGO environment (Papert, 1980).

Taylor indicated that his framework should not be followed slavishly but only as long as it provides useful insights. Interestingly, he considered *toys* as a fourth category, including simulations and games, but felt that they were subsumed under the original three categories. With today's school-age generation and their preoccupation with game consoles, this category may well be the dominant one to apply to the computer *out of* the school.

In 1985, Alessi and Trollip provided another framework for conceptualising the role of computers in education. They suggested the following categories for computer-assisted instruction (CAI): drill, tutorials, games, simulations, and tests. Alessi and Trollip's original categories were aligned to a traditional, expository model of instruction that followed four phases: presenting information; guiding the student; practicing by the student; and assessing student learning (p. 6). Each category of CAI was seen to emphasise one or more of these instructional phases.

In comparing both frameworks, it appears that Alessi and Trollip's categories of drill, tutorial, game, and simulation can be considered as subcategories of tutor, as described in Taylor's scheme. However, electronic *tests* appear to shift the nature of computer use from the learner to teacher and are prescient of the administrative software available to teachers that now exists for planning, monitoring, and evaluating learning environments. Over the last 20 to 30 years, major changes in technology and conceptions of teaching and learning necessitate a re-examination of these earlier taxonomies.

Changes in Technology

Technologies such as CDs, mobile phones, digital cameras, and personal digital assistants are common accessories in the digital home and workplace. The World Wide Web is increasingly becoming part of most schools and classrooms. Online education has a number of benefits over traditional computer-based technologies. Clearly, greater access is provided to those students studying at a distance or unable to mainstream into regular classrooms, as well as those students who wish to learn at their own pace (Santoro, 1995). This type of flexible teaching can enable students to assume greater responsibility for their own learning (Schwier & Misanchuk, 1993; Winn, 1997).

In addition, such an approach can be used to reach a large number of students in a classroom accommodating a broad range of learning styles

(Vargo, 1997; Winn, 1997). Nunan (1996) added that flexible delivery through online methods fosters a culture of self-learning, problem solving, and activity-based learning.

Through such tools as discussion boards and chat rooms, online educational resources enable greater access to synchronous or asynchronous collaborative learning (Mayadas, 1997). Asynchronized learning occurs when a virtual classroom environment can be accessed anywhere at anytime, whereas synchronized learning takes place when learners are connected to an online environment at the same time (Bourne, McMaster, Rieger, & Campbell, 1997). Collaboration is further enhanced because online technologies are also effective in allowing computers with different platforms and browsers to work together in a learning environment, in contrast to other computer technologies that are platform specific (Hosie & Schibeci, 2001). Changes in the way teaching and learning are conceptualized have paralleled changes in technology.

Changes in Pedagogy

Traditional teaching based on behaviorist views of learning is being replaced by inquiry-based teaching, reflecting a constructivist view of learning. According to Elliot, Kratochwill, and Travers (1996), behaviorism focuses on the manipulation of external conditions to modify behaviors that will lead eventually to learning. A behaviorist teaching style in mathematics education tends to stress practices that emphasize rote learning and memorization of formulas, single solutions, and adherence to procedures and drill. Teaching is seen as a matter of enunciating objectives, providing the means to reach those objectives and using constant repetition in class for skill acquisition (Leder, 1994). Wood, Cobb, and Yackel (1991) argued that such approaches lead to passive modes of learning.

In contrast, constructivism claims that knowledge must be actively constructed by learners as they are already “knowing beings” who bring previous knowledge and experience to any learning event. Learning depends on the way the learners interact with situations, beliefs, attitudes, and previous experiences (Biggs & Moore, 1993). For advocates of constructivism, learning is an adaptive and experiential process. Learners tend to look for similarities and differences within their own cognitive schema as they encounter new situations. These contrasts are the end-product of conflictive knowledge looking to be resolved through reorganizing schemes of knowledge (Phillip, 1995). Constructivist teaching strategies include more reflective learning activities, such as problem-solving and inquiry-based learning (Murphy, 1997; Wood et al., 1991).

Re-Examining Frameworks

Given the changes to technology and pedagogy, how relevant and useful for today are the original categories of computer use developed by Taylor and Alessi and Trollip? In Luehrmann's original paper he saw the need for students to learn to use computers as a tool and as a tutee rather than as acting only as the subject of computer tutoring. In his reflection 30 years later, Luehrmann concluded that the "*teaching tool* use is just about the only impact that computers have had on schools" (p.1) —but more often than not, learning *how* to use the tool takes the focus, as opposed to learning *with* the tool.

Fortunately, he argues, the tutor role has come and gone leaving some residual pockets of drill and practice; but he bemoans the fact that the role of the computer as a *tutee* has also disappeared except, perhaps, for those studying computer science. His hope is still that computers will be used as tools to understand learning across all subject areas.

Just as Luehrmann re-examined his framework in 2002, Alessi and Trollip have similarly reflected on their original premise in later editions of their textbook. They now propose an expanded list for the role of computers: tutorials, hypermedia, drills, simulations, games, tools and open-ended learning environments, tests, and web-based learning. This second, expanded list results not only from new technological developments (in particular the Web) but also from new paradigms of teaching and learning (especially constructivism).

Constructivist approaches clearly emphasise a different set of instructional phases and learning outcomes than envisaged in this earlier work. Hypermedia and the use of tools and open ended learning environments are seen as categories that align better with constructivist approaches. It is somewhat curious, however, that Alessi and Trollip suggested web-based learning as another approach, because in their own words, "Web based learning can be combined with any of these other methodologies (for the web is essentially a delivery medium)" (Alessi & Trollip, 2001, p. 12).

These categories are not mutually exclusive and at times overlap each other. For example, a tutorial program can be organized through nodes of information, thus showing some hypermedia-based instruction (HBI) characteristics. Similarly, tools and open-ended learning environments may include some simulation features, while a drill-and-practice activity could take the form of an instructional game. Selecting the appropriate category will depend on the nature and planned outcomes for the mathematics lesson to be taught. The following discussion looks at describing and giving online examples within the context of mathematics education reflecting Alessi and Trollip's later categories.

Drills

Drill-and-practice activities, because of their repetitive nature, still reflect a traditional, behaviorist approach that focuses on mastering basic skills or reviewing material that has been previously learned. A typical drill-and-practice exercise presents learners with a question, followed by response entry and corresponding evaluation of the question and feedback. According to Schwier and Misanchik (1993), for a drill-and-practice activity to be effective, there should be a gradual increase on the "types, amount, and layers of stimuli and feedback presented" (p. 20).

An enriching drill and practice activity should provide opportunities to increase understanding of a mathematical concept as the learner progresses through the activity. Examples of drill-and-practice online sites include Percentage Estimation, Lessons on Order of Operations, Problem Solving, Ratios, Fractions, Percentages, Shape, and Interactive Arithmetic (*Editor's note*: See the Resources section at the end of this article for all website URLs).

Tutorials

Tutorials are one step ahead of drill-and-practice activity, because they not only present information but also guide students through their learning processes. A tutorial usually follows a structure and sequence. The tutorial starts with an introduction to the lesson and information is presented. Next, the learner answers a series of questions and the program evaluates them. Typical responses are “sorry,” “very good,” “try again,” and “right answer is,” among others. In contrast to drill-and-practice approaches, the tutorial will give feedback on the procedure to get the correct answer. The cycle closes when the lesson is terminated, either by the learner or by the program. A summary appears at the close of the lesson.

Tutorials have potential in online interactive learning, because they provide many possibilities to motivate students through multimedia capabilities. Also, a tutorial allows learners to work at their own pace in an individualized mode of instruction and provides many opportunities for reinforcement, correction of mistakes, and elucidation of misunderstandings (Schwier & Misanchuk, 1993). According to Alessi and Trollip (1991) tutorials are effective for “presenting factual information, for learning rules and principles, or for learning problem-solving strategies” (p.17).

One of the advantages of a tutorial is its potential to teach students who do not have a qualified teacher in a particular topic, a problem that occurs in many isolated small schools or when the small number of students in a particular class does not justify the hiring of a specialist teacher (Alessi & Trollip, 1991). Tutorials are also useful in providing individualized instruction but are less effective for collaboration.

There are, however, a number of different ways in which tutorial software can be used. For instance, a tutorial may be employed to support and reinforce classroom instruction, to teach a selected topic, to activate prior knowledge in an area before going to the main topic, or to generate classroom discussion and group work. Tutorials can also provide instruction to students who have missed classes, to review previously encountered topics, or for remediation (Merrill et al., 1992). Tutorials can be combined with other computer devices, such as print, still video, full-motion video, CD-quality audio, computer-generated graphics, animation, and textual overlays. Selected examples of online tutorial sites are Algebra Tutorial, Calculus Tutorial, and What Is the Point?

Games

The next two categories of games and simulations are both goal-oriented activities that provide a multimedia simplification of reality. While using simulators and games, learners encounter a dynamic situation to which they must respond (Linser, Naidu, & Ip, 1999). In a game situation the learner engages in a lose/win situation that requires the practice of skills assumed to be known or in the process of development. Practice is said to facilitate knowledge acquisition (Biggs & Moore, 1993). The artificial environment provided by the software, therefore, motivates the learner through an amusing activity that indirectly provides pedagogical benefits. Examples of instructional games on the WWW include Multiplication Matho, The Hanoi Tower, MathCar Racing, The Number Machine, Worm Hunt, Can You Add Them Up to 24? and Spacey Math.

Simulations

The idea behind simulations is to encourage learning within artificial situations. One of the greatest advantages of simulators is their capacity to represent and connect huge amounts of information through multimedia (Alessi & Trollip, 1991; Gibbons & Fairweather, 1998).

Simulations are similar to games in that they are goal-oriented. They differ from games in that they are not explicitly governed by rules, and there is no competition among the players or against the computer. The major advantage of simulations is their capacity to represent the real-world in circumstances when learning cannot be enacted in real terms; for example, the manipulation of hazardous substances. Computational difficulties, financial constraints in mimicking an activity, timeframe needed to replicate the whole process, or magnitude of the equipment necessary for a certain experiment, each creates unfeasible learning experience.

Teaching mathematics through online simulators requires the teacher to keep a wise balance between personalized scaffolding and autonomous learning. As with all the categories of computer based learning, it is helpful to engage students in teacher and student led discussion before, during, and after the activity. The following links take the reader to online simulation sites: The Broken Calculator, Solve an Equation, Finance Calculator, and Tangrams.

Hypermedia

Hypermedia-based instruction (HBI) is a more complex form of CAI (Ayersman & von Minden, 1995). The basic difference between HBI and CAI is in the organisation of information. While most of the CAI approaches present information on a relatively structured and linear sequence, HBI organises information through a node-and-link structure. Hypermedia approaches combine hypertext and multimedia. Multimedia delivers content using several formats, such as text, sound, graphics, and video that work to reinforce each other (Hall, 2000). Hypertexts are learning environments in which knowledge is represented through a network of nodes of information. Nodes of information are connected through clickable buttons to other nodes, and users control navigation through the nodes. A hypertext has been defined as "a database that has active cross-references and allows the reader to jump to other parts of the database as desired" (Schneiderman & Kearsley 1989, p. 3). The association of nodes on such a nonlinear structure permits a learner to associate a variety of content within an exploratory context.

The non-linear dynamics of HBI empowers students, giving them more autonomy, responsibility, and interactivity with the software (Hall, 2000). Such technical capabilities over traditional CAI approaches permit the learner to build more meaningful connections among texts and information. Ayersman and von Minden (1995) have argued that HBI allows students to acquire more holistic understanding, participate actively in explorative learning, and construct quality knowledge. It has also been claimed that HBI proved successful in reaching a variety of learning styles given its diverse use of media as compared to other traditional forms of instruction (Hall, 2000; Liu & Reed, 1994; Melara, 1996; Summerville, 1999; Weller, Repman, & Rooze, 1994). Some examples in mathematics education occur at Maths Thesaurus, The Symmetry Project, A Maths Dictionary and Matrix: The Virtual Maths Museum.

Tools and Open-Ended Learning Environments

Tools are electronic processes that assist learners in carrying out tasks, such as planning, writing, calculating, drawing, composing, and communicating (Alessi & Trollip, 2001). In mathematics education, tools such as spreadsheets, databases, and graphics packages provide situations for problem solving while also offering open-ended learning environments in which students can investigate such concepts as geometrical and algebraic patterns and relationships. Teachers can use these tools to assist their students in learning mathematics through higher order thinking processes rather than simply learning about the tool. As Luehrmann (2002a, 2002b) indicated, such technological tools may be the predominant approach in many schools. Although many multimedia packages are familiar to mathematics teachers, such as *Tesselmania* (MECC, 1995) and the *Geometer's Sketchpad* (Key Curriculum Press, 1995), their presence on the web is in its early stage. Some of the following examples can be found: Rotating Triangles, Function Plotter, and Create a Graph, JAVA Gallery of Interactive On-Line Geometry, and JavaSketchpad.

CONCLUSION

The previous discussion suggests that Alessi and Trollip's (2001) recent categories of computer use in schools is a helpful framework for classifying web-based mathematics learning activities. As a challenge it would be interesting to see whether these software categories apply equally well for other areas of the curriculum. Although each of these categories facilitates different learning outcomes, simulations, hypermedia and tool-use activities constitute the closest approach to a constructivist view of teaching and learning mathematics. In fact, it could be argued that the sequence of categories from drill, tutorial, game, simulation, and hypermedia to tools and open learning environments mirrors a progression from behaviorist to constructivist pedagogy.

Unfortunately, the frequency of online mathematical tasks reflects an inverse relationship with a great deal less software available at the constructivist end of the continuum. With an ever-increasing reliance on the web, both in the classroom and at home, it is hoped that software developers and educators will become aware of this apparent pattern and consolidate their resources appropriately.

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Resources

A Maths Dictionary - http://www.amathsdictionaryforkids.com/
Algebra Tutorial - http://www.algebrahelp.com/lessons/equationbasics/pgw.htm
Calculus Tutorial - http://www.karlscalculus.org/calc2_1.html
Can You Add Them Up to 24? - http://www.coolmath4kids.com/mathgames/arithmetic24/index.html
Create a Graph - http://nces.ed.gov/nceskids/graphing/bar_pie_data.asp?ChartType=pie
Finance Calculator - http://www.arachnoid.com/lutusp/finance.html
Function Plotter - http://www.karlscalculus.org/cgi-bin/funcplot.pl
Interactive Arithmetic - http://www.scienceacademy.com/BI/index.html
JAVA Gallery of Interactive On-Line Geometry - http://www.geom.uiuc.edu/java/
JavaSketchpad - http://www.keypress.com/sketchpad/JavaSketchpad.html
Lessons on Order of Operations - http://www.mathgoodies.com/lessons/vol7/order_operations.html
MathCar Racing - http://www2.funbrain.com/cgi-bin/osa.cgi?A1=s&A2=2
Maths Thesaurus - http://thesaurus.maths.org/index.html
Matrix: The Virtual Maths Museum - http://www.counton.org/index.html
Multiplication Matho - http://www.aplusmath.com/games/matho/MultMatho.html
Percentage Estimation - http://www.hellam.net/maths2000/percent1.html

Problem Solving, Ratios, Fractions, Percentages, Shape - http://www.teachingtreasures.com.au/maths/maths_level7-pg2.htm
Rotating Triangles - http://www.nrich.maths.org.uk/mathsf/journalf/may99/interact.html
Solve an Equation - http://www.quickmath.com/www02/pages/modules/equations/index.shtml
Spacey Math - http://www.learningplanet.com/sam/sm/index.asp
Tangrams - http://standards.nctm.org/document/eexamples/chap4/4.4/index.htm
The Broken Calculator - http://www.cut-the-knot.com/arithmetic/Calculator.shtml
The Hanoi Tower - http://mathgym.com.au/htdocs/japplets/toh/tower.htm
The Number Machine - http://pbskids.org/cyberchase/games/numbersense/index.html
The Symmetry Project - http://www.adrianbruce.com/Symmetry/
What Is the Point? - http://www5.funbrain.com/cgi-bin/co.cgi
Worm Hunt - http://www.counton.org/index.html

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