Teaching Tessellations to Preservice Teachers Using TesselMania! Deluxe: A Vygotskian Approach

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In the winter of 2000, a class of K-8 preservice teachers were introduced to TesselMania! Deluxe, a software product that visually and manually engages students in the study of transformations but, more specifically, tessellations. Prior to discussing the topic of transformations, students were only slightly familiar with two types of transformations, namely, rotations and reflections. By combining a hands-on activity in which these and other transformations were demonstrated, followed by student exploration of the software, resulted in an unplanned, but welcomed, Vygotskian environment. Students were engaged in rich social interactions, exchanging predictions, and defending their reasoning. Students’ zones of proximal development were bridged, as students both collaborated and practiced independently, gaining a deep understanding of how transformations are used to create tessellations. Technological implications of the software are discussed as well as the potential for K-8 classroom use.

“Wow! Now I get it!” exclaimed a preservice K-8 teacher as she replayed the “morph” feature of TesselMania! Deluxe. Seemingly mesmerized by the tessellation being generated on her monitor, this student had “pieced together” how tessellations are created.

Published by The Learning Company, TesselMania! Deluxe (1997) provides a Vygotskian approach to the study of transformations. The zone of proximal development (ZPD), that is, the distance between a child’s unassisted capability and that child’s capability to perform with support (Wilson,
Teslow, & Taylor, 1993, p. 81) is bridged by the opportunities for exploration and additional practice offered by *TesselMania! Deluxe*. As students discover and visually experience how tessellations are created through a series of transformations, students are enticed to interact with classmates as they are dazzled by the software’s colorful animations. Thus, *TesselMania! Deluxe*’s visually dynamic approach to the teaching and learning of tessellations embraces the spirit of Vygotsky, as it stimulates encounters between and among the teacher and students, bridging the students’ ZPD by providing meaningful experiences.

**WHY STUDY TESSELLATIONS?**

Derived from the Latin word, *tessella*, which were the tiny pieces of stone used to make Roman mosaics, a tessellation, or tiling, is a pattern made from one or more shapes that fit together with no gaps or overlaps. These shapes fit together by performing one or more transformations. In its *Standards for Pre-K-2*, the National Council of Teachers of Mathematics (NCTM, 2000) states that transformations are “an important part of spatial learning” as they help students “become conscious of the motions and encourage them to predict the results of changing a shape’s position or orientation, but not its size” (p. 99). For students in grades 6-8, the study of transformations not only builds students’ spatial reasoning skills as students analyze and explain mathematical relationships, but also offers a lens through which students can investigate and interpret geometric objects as they explore similarity, symmetry, and congruence properties (NCTM, 2000). Additionally, experiences with transformations, in particular, tessellations, enable elementary students to integrate mathematics and art (O’Daffer, Charles, Cooney, Dossey, & Schielack, 1998) and provide a “genuine, natural multicultural connection in the context of art” (Hatfield, Edwards, Bitter, & Morrow, 2000).

Therefore, the study of tessellations in the K-6 classroom has many profound implications, students not only engage in a geometric investigation of transformations and polygons, but they can also develop an understanding and appreciation for how mathematics is connected to their world. Students unknowingly encounter tessellations almost everyday in their lives, from the tile floor they walk on to the brick walls enveloping their school. The artwork of the Dutch artist, M.C. Escher, also provides another connection between the mathematics of tessellations and real life. Thus, the study of tessellations, while providing a constructivist environment for learning, forges a natural bridge for collaboration among art, history, and mathematics teachers.
MANUALLY CREATING TESSELLATIONS

In the winter of 2000, a class of preservice K-8 teachers was introduced to *TesselMania! Deluxe*. Prior to exploring the software, students were first engaged in a hands-on paper cutting and tracing activity in which they created their own tessellations using three different types of transformations. Included in the software’s manual are excellent step-by-step instructions for manually creating tessellations using a translation, rotation, and a glide reflection.

Students were placed in groups of four, sharing paper, scissors, rulers, and tape. Students began by cutting a two-inch square from a 4’ x 6’ index card. Students were asked to follow the instructor’s lead as they first drew a free-form line on their square from one corner to the other (Figure 1). Using their scissors, students cut along the line and then slid this piece downward to the opposite side of the square (Figure 2), securing it with a piece of tape. Students were informed that they just performed a translation, or a slide, which is a particular type of transformation. In a similar fashion, students performed another translation by using the remaining two sides of the square (Figure 3).

Figure 1. Draw free-form line from one corner to the other

Figure 2. Translate piece downwards
Students were then asked to trace their “tile” onto an 8 1/2’ x 11’ piece of paper. Students were instructed to retrace the tile again and again, in an attempt to create a pattern without any gaps or overlaps, that is, a tessellation (Figure 4). Students appeared to complete this task with relative ease, simply sliding their tile across the page and down. Many students commented how their tile resembled a jigsaw puzzle piece and thus, they felt engaged in the familiar task of completing a jigsaw puzzle.

Students repeated this same activity, this time creating a new tessellation using two rotations. Creating a tessellation using a tile that has undergone two rotations was much more challenging for students, as they had to properly rotate the tile, orienting it such that it fit perfectly adjacent to the
previously traced tile, leaving no gaps or overlaps. One of the students immediately objected, explaining that her shape would “never tessellate” because it was “too weird looking” (Figure 5). She was encouraged to try, pointing out that other students in her group were able to tessellate their tile. She unsuccessfully experimented, positioning her tile in various orientations, even flipping it over at one point. A “Vygotskian moment” occurred next as other students in her group leapt to her assistance, commanding her to “Keep turning it!” (These students recalled that when generating the first tessellation, the tile was continually slid, or translated across the paper, as this was the very transformation used to create the tile. Therefore, in the case of a rotation, in order to create the corresponding tessellation, the tile needed to be rotated.) Through the assistance and support of her peers, this student was able to tessellate her tile and she smiled proudly (Figure 6). She commented, “That’s right. I’ve got to turn it, not slide it.” For this student, as Vygotsky notes, learning occurred twice: initially at the social level in collaboration with other people, and then on the individual level where learning was internalized (Vygotsky, 1978, p. 57).

Figure 5. Tile created using two rotations

In the final activity, students were instructed how to create a tessellation using a glide reflection. The instructor thoroughly enjoyed watching the aforementioned student quickly and confidently generate her tessellation, gliding her tile, reflecting it across her paper, and then insisting on helping the other students in her group with their tessellations. Clearly, this student’s prior interaction and experience with her peers facilitated this student bridging her ZPD to reach a heightened state of understanding.
Wilson et al., (1993) extend methods of bridging the ZPD to include technological tools and devices, as technological tools “can provide both context and support for meaningful problem-solving activities” (p. 81). After students created tessellations manually, students began exploring the use of transformations to create tessellations using the technological tool, *Tes selMania! Deluxe*. When using the software to create a tessellation, transformations are performed automatically, allowing students to instantly see the results of their modifications to the original tile. This animation mirrored the hands-on activity in which the students previously engaged, thus reinforcing and deepening students’ understanding of the effects of transformations. Thus, the ability of the software to mimic and further extend the visual effects of transformations provided a Vygotskian perspective to learning, as the exploration of the software took students “beyond their zone of comfort” so that they could “maximize learning and bridge their ZPD’s” (Taylor, 1993, p. 14).

Unlike the hands-on activity in which students were limited to performing only two transformations of the same type per tile, using *Tes selMania! Deluxe*, students can perform a myriad of transformations and designate the particular type(s) of transformation to be performed. After transforming a tile, students can then use one of the “magic buttons” and watch the tile dynamically tessellate on their monitor. Additionally, students can play and
replay the sequence of transformations used in creating the tile and/or the
tessellation, varying the animation speed to one’s liking. Several students
were observed using this feature repeatedly as the animation reinforced the
resulting effects of transformations, deepening their understanding. This
was evidenced by their comments such as, “Cool! I totally get it now!” and
“That’s what we just did!”

When creating a tessellation, the software keeps a record of the Heesch
type used in creating the tessellation. Named after the classification system
developed by the German mathematician Heinrich Heesch, a tessellation’s
Heesch type is a lettered code that represents the transformations used to
create the tile. In the first hands-on activity previously described, the Heesch
type of the tile created would be TTTT, indicating that two translations were
performed on a square tile to create the new tile. One of the 14 “classroom
ideas” listed in the *TesselMania! Deluxe* manual describes a challenging ac-
tivity in which students use their newly gained understanding of transforma-
tions to analyze a tessellation, working backwards to identify how it was cre-
ated, using its Heesch type. This activity embodies the spirit of Vygotsky’s
work, as the teacher, by using the software can “initiate the solution and ask
the child to finish it or offer leading questions” (Vygotsky, 1978, p. 86).

Another classroom idea appearing in the *TesselMania! Deluxe* manual,
indicative of Vygotsky’s theory of social interaction, directs students to first
stamp their name onto a tile using the stamp tool and then predict how their
name will appear once it has undergone a series of transformations. Students
worked in pairs, taking turns, when completing this activity. The noise level
in the room increased as the students made predictions and defended their
reasoning. Laughter and sighs were abound as students tessellated the tile,
saw the outcome, and compared it to their predictions.

**TECHNOLOGICAL IMPLICATIONS**

Due to time constraints, the instructor demonstrated *TesselMania! De-
luxe* to a class of preservice K-8 teachers prior to engaging them in the
hands-on, paper cutting and tracing activity. Although the students seemed
to be enticed by the colorful visual displays and the software’s user-friendli-
ness, students simply did not grasp what was happening mathematically as
they pulled and tugged on the sides of a polygon using transformations to
generate a tessellation on their screens. Looks of confusion and frustration,
and students whispering, clearly seeking help and further explanations from
their classmates was observed. Students were quickly reassured that the
upcoming hands-on paper cutting activity in which they were about to en-
gage would “make sense of what they were viewing on their monitors.” While this software did promote social interactions, they were not the social interactions that Vygotsky spoke of.

Thus, teachers were strongly cautioned in relying solely on this software as a means for teaching tessellations and transformations, as it would be irresponsible. This technology is a lovely supplement to the study of tessellations, but certainly not a replacement. The hands-on paper cutting component is a must and is the author’s strongest recommendation is to engage students in this activity first. This sequence of activities would then provide students with a true Vygotskian learning experience.

**CLOSING**

In his works, Vygotsky highlighted the dialogue that occurs between mother and child, or between student and teacher. Similar to Wilson et al., (1993), the author offers that student-technology interactions are similar to these aforementioned interactions and therefore are instrumental in a student’s level of potential development. Collins (1991) also emphasized the role of technology in creating learning environments and performance supports.

The excitement, motivation, and interaction exhibited on the part of the students when using *TesselMania! Deluxe* was unforgettable. Students’ creativity as they transformed a blank tile into animals, people, and other fun shapes using the paint and stamp tools (Figure 7) were amazing. By integrating *TesselMania! Deluxe* into the teaching and learning of tessellations, students were rewarded with a wealth of examples of tessellations that could not have been provided otherwise while simultaneously learning about transformations. Additionally, students were afforded the opportunity to experiment, discover patterns and relationships, engage in problem-solving, and test and verify conjectures, all within a cooperative learning environment.

Given the fruitful experiences observed on the part of the preservice K-8 students as they interacted with *TesselMania! Deluxe*, elementary and middle school children would benefit from this Vygotskian exploration of tessellations, moving students forward into their ZPD. In guiding these preservice teachers from a collaborative hands-on experience into a collaborative software exploration, students developed a deeper understanding of transformations and tessellations while creating real-life connections to the world of patterns and art. Based on the feedback received from this group of K-8 preservice teachers, this rich, interactive lesson provided these individuals with the confidence and expertise to teach tessellations to young children, exemplifying Vygotsky’s words, “…what children can do with as-
sistance today they will be able to do by themselves tomorrow” (Vygotsky, 1978, p. 87).

Figure 7. Samples of students’ tessellations
References


